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Influence of Radiation on Double Diffusive Flow in a Porous Medium Embedded With Vertical Plate

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Keywords

Porous Medium •
Darcy Flow •
Vertical Plate •
Double Diffusion •
Radiation •
Natural Convection •
FEM.

Abstract

Coupled heat and mass transfer in a saturated porous media supported by a vertical plate is analyzed. The fluid inside the porous medium is assumed to be governed by Darcy law. The combined effect of natural convection and radiation on heat and mass transfer inside the medium is investigated. Finite element method is applied to solve the governing equations. Influence of radiation parameter, Lewis number and buoyancy ratio on Nusselt number and Sherwood number is investigated. Both aiding and opposing flow is considered for a wide range of radiation parameter, Lewis number and Rayleigh number.

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Nomenclature

C_p	Specific heat
C	Species concentration
\bar{C}	Species concentration (non-dimensional)
D	Mass diffusivity
g	Gravitational acceleration
h	Convective heat transfer coefficient

k	Thermal conductivity
K	Permeability of porous media
L_{ref}	Reference length
Le	Lewis number
M	Shape function
N	Buoyancy ratio
Nu	Local Nusselt number
p	Pressure
q_r	Radiation flux
x, y	Cartesian co-ordinates
\bar{x}, \bar{y}	Non dimensional co-ordinates
Ra	Rayleigh number
Rd	Radiation parameter
T	Temperature
\bar{T}	Non-dimensional Temperature
Sh	Local Sherwood number
u	Velocity in r direction
v	Velocity in y direction
<i>Greek Symbols</i>	
α	Thermal diffusivity
β_C	Coefficient of concentration expansion
β_T	Coefficient of thermal expansion
β_R	Rosseland extinction coefficient
ρ	Density
ν, μ	Coefficient of kinematic and dynamic viscosity respectively
σ	Stephan Boltzmann constant
ψ	Stream function
$\bar{\psi}$	Non-dimensional Stream function
<i>Subscripts</i>	
w	wall
∞	Conditions at infinity

1. Introduction

Analysis of fluid flow and heat transfer in a fluid saturated porous medium has become a separate topic of research in last two decades. Flow through porous medium is a subject of high interest in a wide range of engineering disciplines such as chemical engineering, mechanical engineering, soil science, bio-engineering etc. to name but a few. The determination of heat and mass transfer in a porous body has many practical applications such as migration of moisture through the air contained in fibrous insulation, heat exchangers, geothermal energy extraction, chemical contaminants through soil, disposal of nuclear waste, solar heating systems, packed bed catalytic converters etc. A good insight into the subject is given by Nield and Bejan (1999), Ingham and Pop (1998), Vafai (2000), Pop and Ingham (2001), Bejan and Kraus (2003). A comprehensive study of the natural convection heat and mass transfer inside a porous layer is presented

by Osvar and Bejan (1995). Bejan and Khair (1985) have addressed the problem of buoyancy induced heat and mass transfer from a vertical plate embedded in a porous medium. Angirasa et al. (1997) have analyzed the effect of heat and mass transfer by natural convection in a fluid saturated porous medium by using alternating direction implicit scheme. Nakayama and Hossain (1995) have presented an integral treatment for combined heat and mass transfer by natural convection in a porous medium. Double dispersion effects on natural convection heat and mass transfer by considering Forchheimer extension to Darcy flow, is studied by El-Amin (2004). Ching (2000) has found that the local Nusselt and Sherwood number increases with the increase of amplitude-wavelength ratio in a wavy surface in a porous medium adjacent to vertical plate. Radiation plays a vital role when convection is relatively small and thus cannot be neglected. The effect of radiation on mixed convection from a vertical flat plate in a saturated porous medium is investigated by Bakier (2001) using fourth order Runge-Kutta method. Hossain and Pop (1997) have investigated the effect of radiation on Darcy free convection flow along an inclined surface in porous media and presented results for non-dimensional temperature, local Nusselt number etc. Hossain and Pop (2000) have also studied the radiation effect on free convection over a vertical flat plate with high porosity using local non-similarity and implicit finite difference methods. Raptis (1998) has investigated the heat transfer behavior of vertical plate in porous medium subjected to constant suction velocity. Badruddin et al. (2006a, 2006b) studied the free convection and radiation for varying boundary conditions about a vertical plate.

Most of the previous studies have concentrated on heat and mass transfer by natural convection in a porous medium. Although various aspects of heat and mass transfer are addressed by many researchers but the radiation effect on heat and mass transfer has not received much attention. The present work is devoted to investigate the combined effect of natural convection and radiation on heat and mass transfer in a fluid saturated porous medium supported by a vertical plate subjected to uniform wall temperature and uniform wall concentration. The heat and mass transfer is analyzed for both, aiding and opposing flow.

2. Analysis

A vertical plate adjacent to the saturated porous medium is considered. The schematic of problem considered is shown in figure 1. The x coordinate is taken normal to the plate whereas y coordinate is parallel to vertical plate. The Darcy model is assumed to hold good for flow in porous medium. Following assumptions are applied;

- Porous medium is saturated with fluid.
- The radiation effect is negligible in y direction
- The fluid and medium are in local thermal equilibrium everywhere inside the medium
- The porous medium is isotropic and homogeneous.
- Fluid properties are constant except the variation of density which allows natural convection to take place.

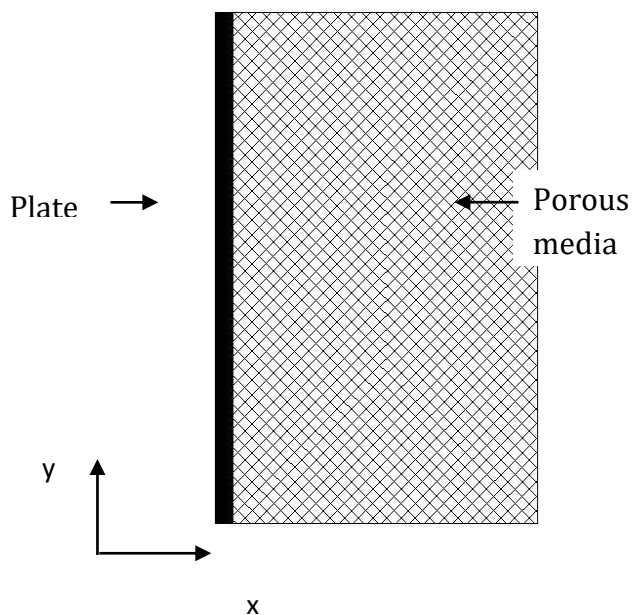


Figure 1: Schematic of porous media supported by vertical plate

Under above conditions the governing equations are given as:

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Using Darcy law for flow in porous media

$$u = -\frac{K}{\mu} \frac{\partial p}{\partial x} \quad (2)$$

$$v = -\frac{K}{\mu} \left(\frac{\partial p}{\partial y} + \rho g \right) \quad (3)$$

Energy equation

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial x} \quad (4)$$

$$\rho = \rho_\infty [1 - \beta_T (T - T_\infty) - \beta_C (C - C_\infty)] \quad (5)$$

Species concentration

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) \quad (6)$$

Invoking Rossel and approximation for radiation [12]

$$q_r = - \frac{4\sigma}{3\beta_R} \frac{\partial T^4}{\partial x} \quad (7)$$

The term T^4 can be expanded about T_∞ using Taylor series [15] as:

$$T^4 \approx 4TT_\infty^3 - 3T_\infty^4 \quad (8)$$

The boundary conditions

$$\begin{aligned} \text{at } x=0, \quad u=0, \quad T=T_w, \quad C=C_w \\ \text{as } x \rightarrow \infty, \quad u \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \end{aligned} \quad (9)$$

Introducing stream function ψ as:

$$u = \frac{\partial \psi}{\partial y} \quad (10)$$

$$v = - \frac{\partial \psi}{\partial x} \quad (11)$$

The following parameters have been used for non-dimensionalisation of the governing equations.

$$\begin{aligned} \text{Non-dimensional width} \quad \bar{x} &= \frac{x}{L_{ref}} \\ \text{Non-dimensional height} \quad \bar{y} &= \frac{y}{L_{ref}} \\ \text{Non-dimensional stream function} \quad \bar{\psi} &= \frac{\psi}{\alpha} \\ \text{Non-dimensional temperature} \quad \bar{T} &= \frac{(T - T_\infty)}{(T_w - T_\infty)} \\ \text{Non-dimensional concentration} \quad \bar{C} &= \frac{(C - C_\infty)}{(C_w - C_\infty)} \end{aligned} \quad (12)$$

$$Rd = \frac{4\sigma T_{\infty}^3}{\beta_R k}$$

Radiation parameter

$$Ra = \frac{g\beta_T \Delta T K L_{ref}}{\nu \alpha}$$

Rayleigh Number

$$Le = \frac{\alpha}{D}$$

Lewis number

$$N = \left(\frac{\beta_c \Delta C}{\beta_T \Delta T} \right)$$

Buoyancy ratio

Making use of equations 7-8, 10-12 and having done necessary mathematical operations yields:

$$\frac{\partial^2 \bar{\psi}}{\partial x^2} + \frac{\partial^2 \bar{\psi}}{\partial y^2} = -Ra \left[\frac{\partial \bar{T}}{\partial x} + N \frac{\partial \bar{C}}{\partial x} \right] \quad (13)$$

$$\frac{\partial \bar{\psi}}{\partial y} \frac{\partial \bar{T}}{\partial x} - \frac{\partial \bar{\psi}}{\partial x} \frac{\partial \bar{T}}{\partial y} = \left(\left(1 + \frac{4Rd}{3} \right) \frac{\partial^2 \bar{T}}{\partial x^2} + \frac{\partial^2 \bar{T}}{\partial y^2} \right) \quad (14)$$

$$\frac{\partial \bar{\psi}}{\partial y} \frac{\partial \bar{C}}{\partial x} - \frac{\partial \bar{\psi}}{\partial x} \frac{\partial \bar{C}}{\partial y} = \frac{1}{Le} \left(\frac{\partial^2 \bar{C}}{\partial x^2} + \frac{\partial^2 \bar{C}}{\partial y^2} \right) \quad (15)$$

Subjected to boundary conditions

$$\begin{aligned} \text{at } \bar{x} = 0, \quad \bar{\psi} = 0, \quad \bar{T} = 1, \quad \bar{C} = 1 \\ \text{as } \bar{x} \rightarrow \infty, \quad \frac{\partial \bar{\psi}}{\partial \bar{x}} = 0, \quad \bar{T} \rightarrow 0, \quad \bar{C} \rightarrow 0 \end{aligned} \quad (16)$$

3. Numerical method

Equations (13)-(15) are coupled partial differential equations in the non-dimensional form, subjected to boundary conditions (16). Equations (13)-(15) are to be solved in order to predict the heat and mass transfer behavior in the porous medium. In the present case, these equations are solved by using finite element method. Galerkin method is applied to convert the partial differential equations into matrix form of equations. A simple 3 noded triangular element is used to represent the variations of temperature, stream function and concentration gradient inside the element. \bar{T} , $\bar{\psi}$ and \bar{C} vary inside the element and can be expressed as:

$$\begin{aligned}\bar{T} &= \bar{T}_1 M_1 + \bar{T}_2 M_2 + \bar{T}_3 M_3 \\ \bar{\psi} &= \bar{\psi}_1 M_1 + \bar{\psi}_2 M_2 + \bar{\psi}_3 M_3 \\ \bar{C} &= \bar{C}_1 M_1 + \bar{C}_2 M_2 + \bar{C}_3 M_3\end{aligned}\quad (17)$$

where M_1, M_2, M_3 are the shape functions given as:

$$M_i = \frac{a_i + b_i x + c_i y}{2A}, \quad i = 1, 2, 3 \quad (18)$$

Details of FEM formulations can be obtained from Segerland LJ (1982), Lewis RW et al. (2004). First, equations (13)-(15) are converted into matrix form of equations for an element. The coupled matrix equations for elements are assembled to get the global matrix for the whole domain, which is solved iteratively, to obtain \bar{T} , $\bar{\psi}$ and \bar{C} in the porous medium. In order to get accurate results, tolerance level of solution for \bar{T} , $\bar{\psi}$ and \bar{C} are set at 10^{-5} , 10^{-9} 10^{-6} respectively. Element size in the domain varies such that the small sized large number of elements are located near the wall where large variations in \bar{T} , $\bar{\psi}$ and \bar{C} are expected. Sufficiently dense mesh is chosen to make the solution mesh invariant. The computations are carried out on high-end computer. Table 1 shows the $Nu/Ra^{1/2}$ for different mesh sizes. It can be seen from table 1 that the variation in $Nu/Ra^{1/2}$ is very small (0.16%) when element size is changed from 2000 to 3000. The computational time required to solve mesh size of 3000 elements is very large compared to that of 2000 elements. Thus mesh size of 2000 elements is selected in the present study. For many practical cases, even a mesh of 1000 elements is adequate provided a dense mesh is used near the wall since there is not much variations (0.5%) in $Nu/Ra^{1/2}$ when mesh size is increased from 1000 to 2000.

The heat transfer rate at plate surface is given by:

$$q_w = - \left[\left\{ k + \frac{16 \sigma T_\infty^3}{3\beta_R} \right\} \frac{\partial T}{\partial x} \right]_{x=0} \quad (19)$$

The local Nusselt number is expressed as:

$$Nu = \frac{hy}{k} = \frac{q_w y}{k (T_w - T_\infty)} \quad (20)$$

$$Nu = - \left[\left(1 + \frac{4Rd}{3} \right) \frac{\partial \bar{T}}{\partial \bar{x}} \right]_{\bar{x}=0} \quad (21)$$

The local Sherwood number is expressed as:

$$Sh = -\frac{\partial \bar{C}}{\partial \bar{x}} \Big|_{\bar{x}=0} \quad (22)$$

Table 1: Nu variation with mesh size

Sr No	No of elements	$Nu / Ra^{1/2}$
1	1000	0.4415
2	2000	0.4393
3	3000	0.4386

3. Results and discussion

The surface temperature of the plate is maintained at T_w higher than the ambient temperature T_∞ . Similarly, the chemical species concentration varies from C_w at the vertical plate to C_∞ at ambient condition. In order to predict the heat and mass transfer behavior, results are obtained in terms of Nusselt number and Sherwood number. The study is carried out for a wide range of buoyancy ratio N , Lewis number Le , Rayleigh number Ra and radiation parameter Rd . The buoyancy ratio is a measure of relative importance of mass and thermal diffusion. N is -ve when thermal buoyant force acts vertically upward and molecular buoyant force acts downward, making the flow as thermally opposing. N is zero for thermally driven flow and positive if both the forces i.e. thermal buoyant and molecular buoyant are acting upwards assisting each other and thus flow is thermally assisted.

In order to verify the accuracy of the present method, results are compared with those available in the literature Yih KA (1999). Table 2 shows the comparison. It is obvious from table 2 that the present method compares well with the available literature.

Table 2: Comparison of present method

N	Le	$Nu / Ra^{1/2}$		$Sh / Ra^{1/2}$	
		Yih [18]	Present	Yih [18]	Present
4	1	0.9922	0.9455	0.9922	0.9455
	10	0.6810	0.6306	3.2897	3.2414
	100	0.5207	0.4889	10.5203	10.3949
1	1	0.6276	0.6039	0.6276	0.6039

	10	0.5214	0.5046	2.2020	2.1643
	100	0.4700	0.4602	7.1389	7.0579
	1	0.4437	0.4442	0.4437	0.4442
0	10	0.4437	0.4442	1.6803	1.6534
	100	0.4437	0.4442	5.5445	5.4731

Figure 2 shows the Isotherms, isoconcentration lines and streamlines for opposing and aiding flows. It can be seen that the thermal boundary layer is thicker for opposing flow as compared to that of aiding flow (figure 2a). The concentration layer is thinner for aiding flow which gives rise to high concentration gradient near the plate (figure 2b). The streamlines move towards the vertical plate when N is increased, which indicates that the convection heat transfer increases with aiding flow. The effect of Lewis number and radiation parameter on the Isotherms, isoconcentration lines and streamlines is depicted in figure 3. The increase in Lewis number leads to increase in the thermal boundary layer inside the porous medium. However the concentration boundary layer shrinks with the increased Lewis number. The radiation parameter also increases the thermal boundary layer.

Figure 4 shows Nu and Sh for different values of buoyancy ratio and radiation parameter at $Ra=100$ and $Le=10$. It may be noted that the zero value of Rd corresponds to the pure natural convection without any radiation. The Nu and Sh are lower for opposing flow ($N = -1$) as compared to aiding flow ($N = 4$ and 10). The difference between Nu for aiding and opposing flow is maximum for pure natural convection. The radiation parameter reduces the effect of buoyancy ratio. For a given radiation parameter, the Nu is higher for higher buoyancy ratio. For Rd greater than 7, there is no significant change in Nu with respect to N . In general, Nu increases with increase in Rd and effect of buoyancy ratio diminishes at higher radiation parameter. The Sherwood number is not much affected by Rd . It is observed that the Sh increases slightly with increase in Rd . At $N = 10$, the Sh increased by 1.5% when Rd is varied from 0 to 10.

Figure 5 shows the effect of radiation parameter and Lewis number on Nu and Sh at $Ra = 100$ and $N = 4$. The effect of Lewis number also vanishes at higher Rd . A marginal increase in Sherwood number is observed when Rd is varied from 0 to 10. Sh is increased by 0.8% at $Le=100$, when Rd is varied from 0 to 10. In general, for a given Rd , the Sherwood number increases with increase in Le .

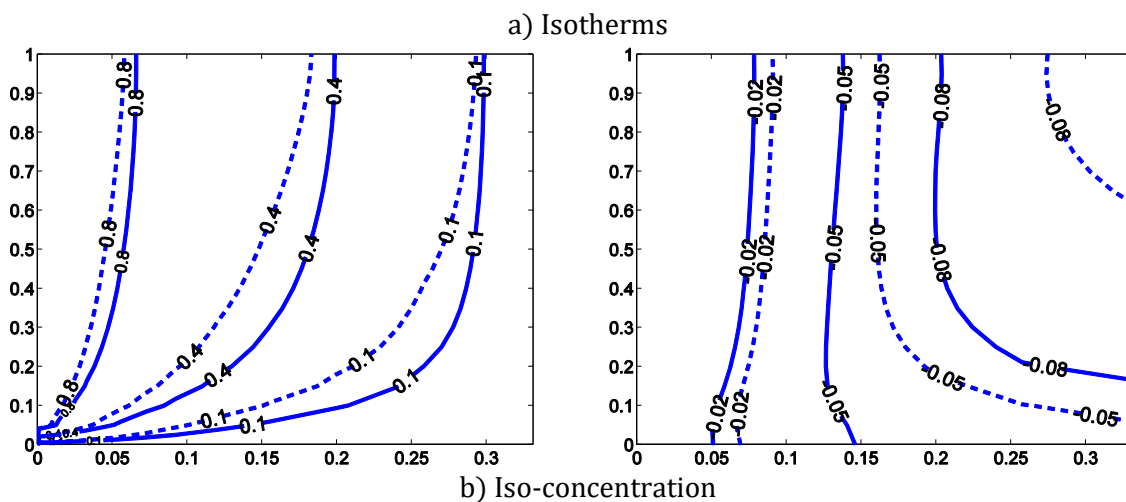
Figure 6 shows Nu and Sh for different values of buoyancy ratio when Lewis number is varied at $Ra=100$ and $Rd=1$. The Nusselt number decreases with increase of Lewis number. This happens because of the reason that the thermal diffusivity increases with increased Lewis number, which in turn is the indication of increased thermal conductivity. Thus increased thermal conductivity leads to reduction in Nusselt number. For a given Lewis number, Nusselt number is higher for assisting flow as compared to opposing flow. The effect of N on Nusselt number vanishes at large value of Lewis number. Sherwood number increases with the increase in Lewis number. This is due to the fact that higher Lewis number reduces the concentration boundary layer thus creating the high species concentration gradient. The effect of Lewis

number is more pronounced on Sherwood number than on the Nusselt number. The thermal boundary layer decreases with increased Rayleigh number as shown in figure 7a, which in turn increases the temperature gradient. This happens because; the increased Rayleigh number is associated with lower thermal diffusivity which in turn leads to lower thermal conductivity. The low thermal conductivity of the medium creates a thinner thermal boundary layer. The species concentration layer becomes thinner at higher Rayleigh number (figure 7b) and thus Sherwood number increases. The streamlines tend to distort at higher Rayleigh number as shown in figure 7c.

Figure 8 shows the dependency of Nu and Sh on buoyancy ratio, Rayleigh number and Lewis number for $Rd=1$. The Nusselt number increases with increase of Rayleigh number as expected. This is because increased Rayleigh number is associated with higher convective heat transfer, which in turn leads to increased Nu . Increased Rayleigh number also increases the Sherwood number. When Rayleigh number increases the concentration boundary layer becomes thinner and thus increasing the species concentration gradient. The increased Rayleigh number and radiation parameter enhances the Nu and Sh as shown in Figure 9.

The non-dimensional temperature gradient is shown in figure 10. It can be seen that the thermal boundary layer increases with increase in Lewis number thus reducing the temperature gradient at the plate. The radiation parameter Rd , reduces the temperature gradient for a given Lewis number. Figure 11 shows the temperature profile for different values of N and Rd . The thermal boundary layer decreases with increase in N thus increasing temperature gradient at the plate. As in previous case, it can be seen that the thermal boundary layer increases due in increase in radiation parameter.

Figure 12 shows the species concentration profile with respect to Lewis number. Species concentration gradient increases with increase in Lewis number. It can be seen that the radiation parameter does not have significant effect on the species concentration profile. Figure 13 shows the species concentration profile at various values of N . The concentration layer thickness reduces with increase in N thus species concentration gradient and the Sherwood number increases with increased buoyancy ratio. As in previous case, radiation parameter does not have much effect on concentration gradient especially at $Rd=0$.



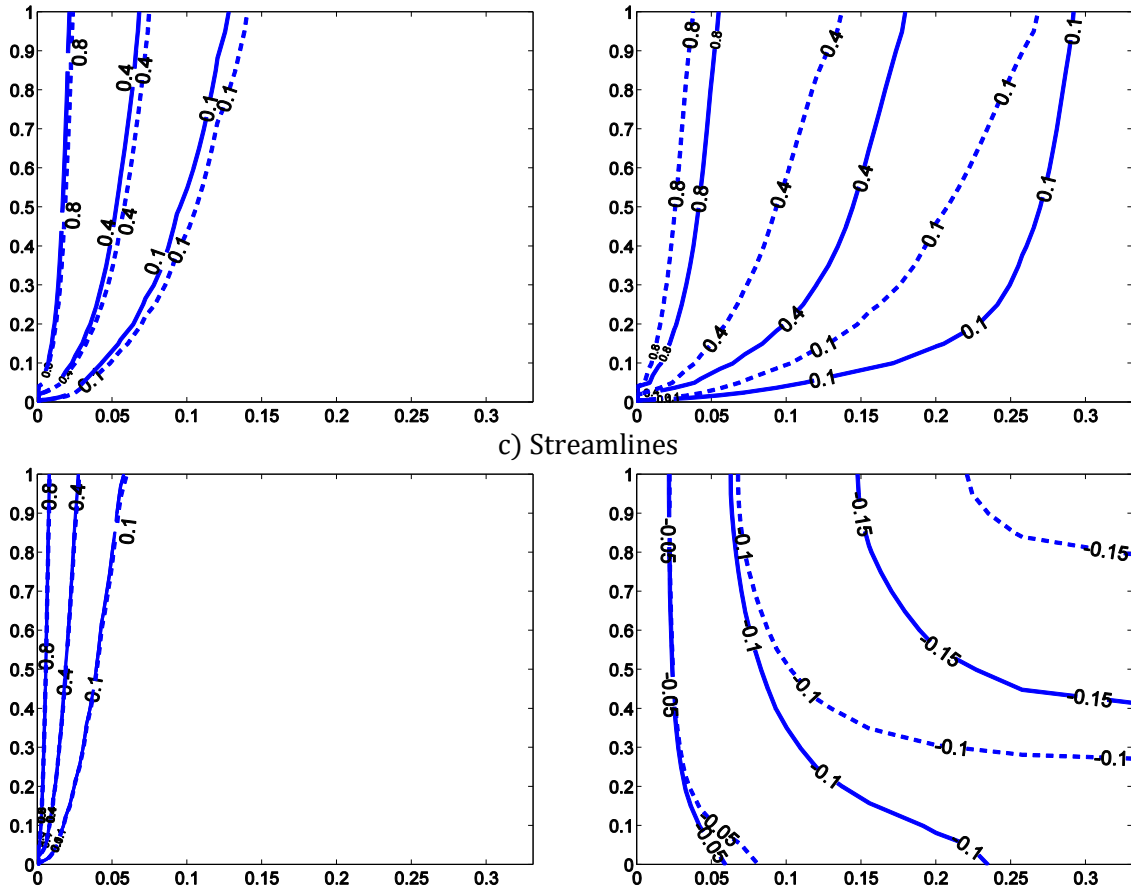
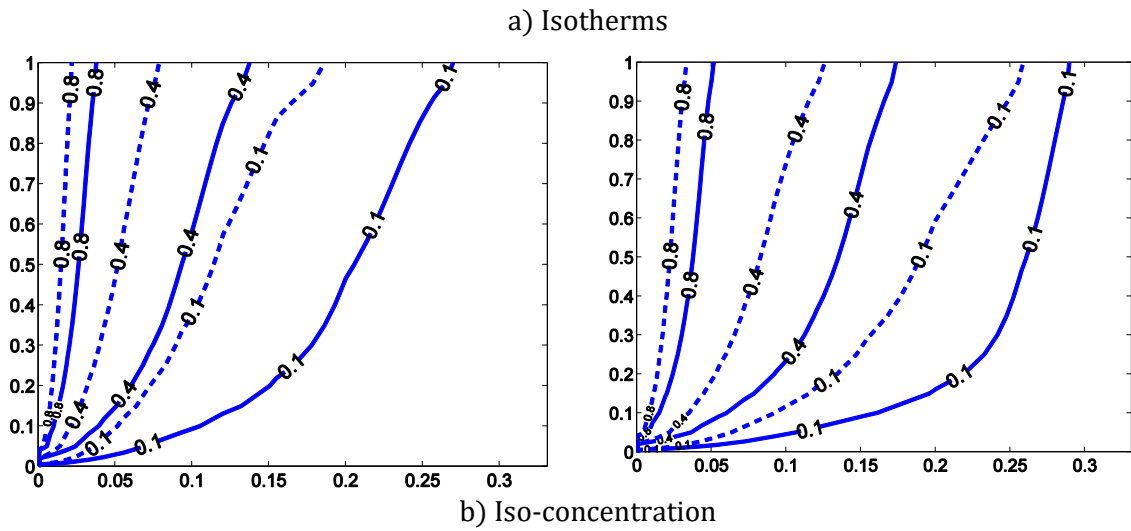


Figure 2: a) Isotherms b) Iso-concentration and c) Streamlines
 Left $N = -1$, Right $N = 2$; Dotted line $Rd = 0$, Solid line $Rd = 1$



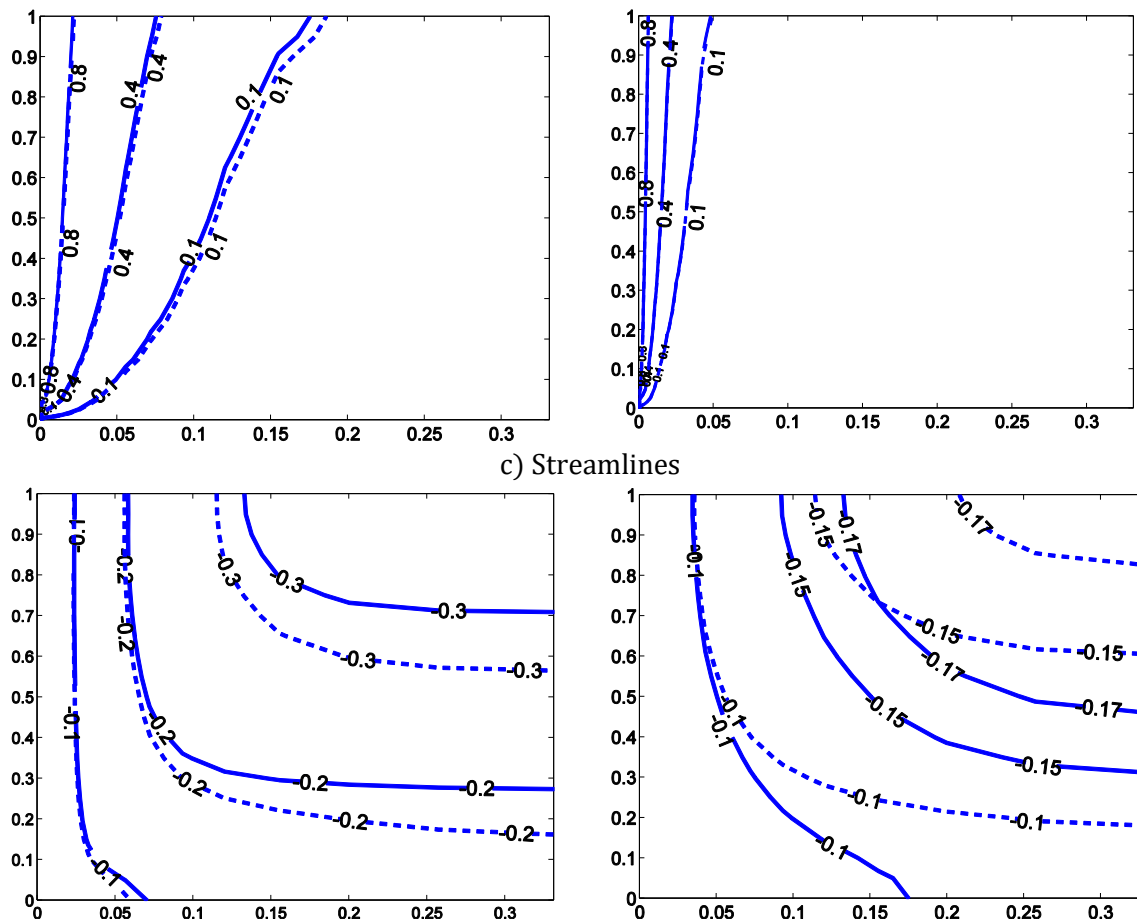


Figure 3: a) Isotherms, b) Iso-concentration and c) Streamlines
 Left $Le = 1$, Right $Le = 10$; Dotted line $Rd = 0$, Solid line $Rd = 1$

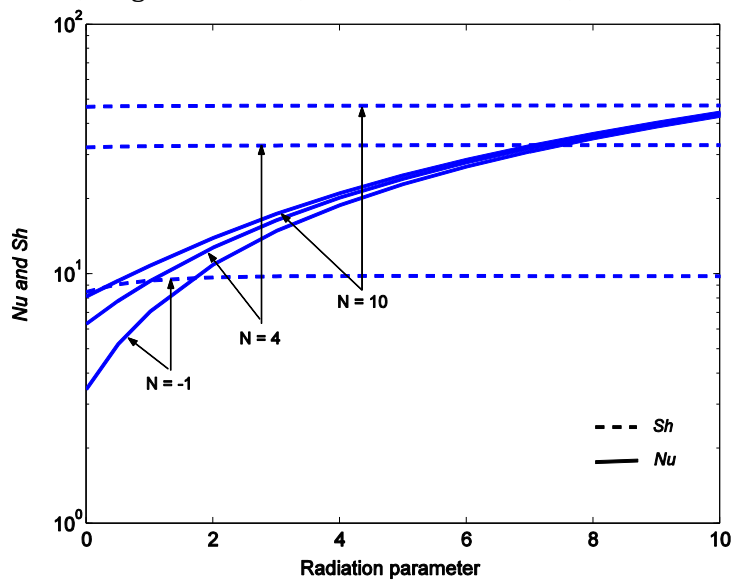


Figure 4: Nu and Sh for different values of N and Rd

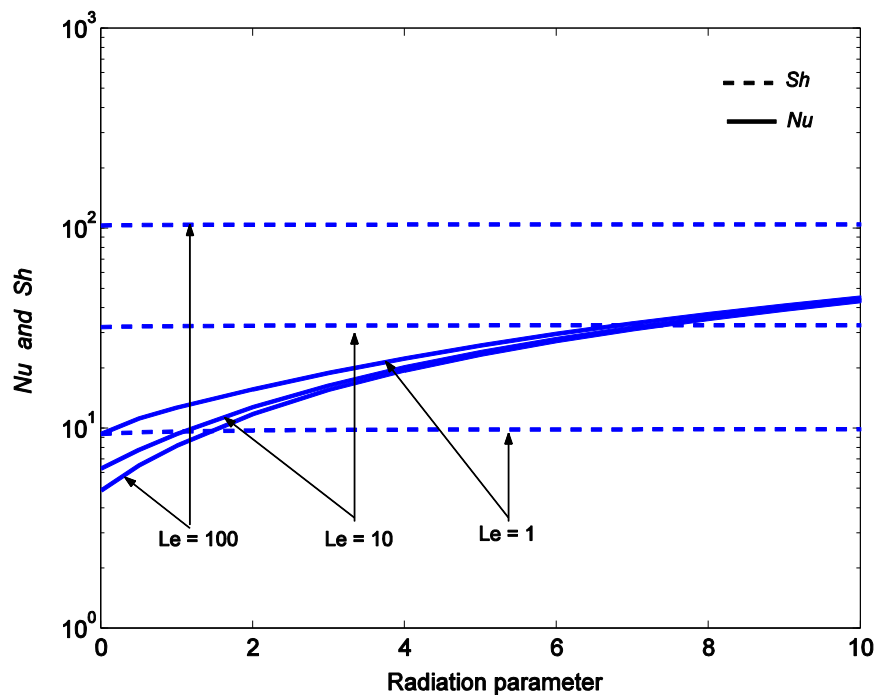


Figure 5: Nu and Sh for different values of Le and Rd

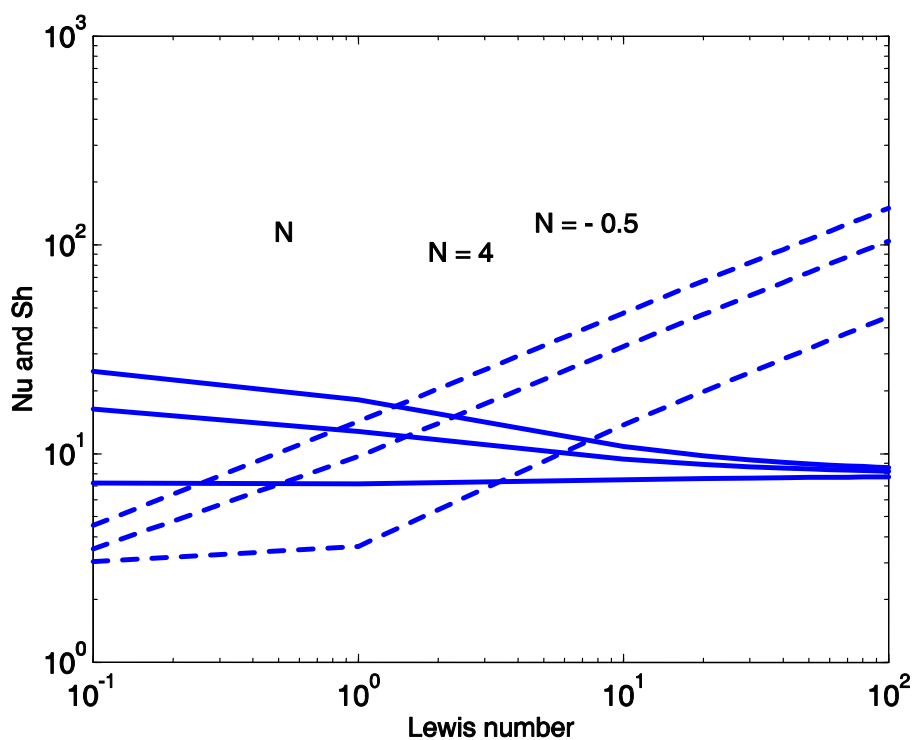


Figure 6: Nu and Sh for different values of N and Le.

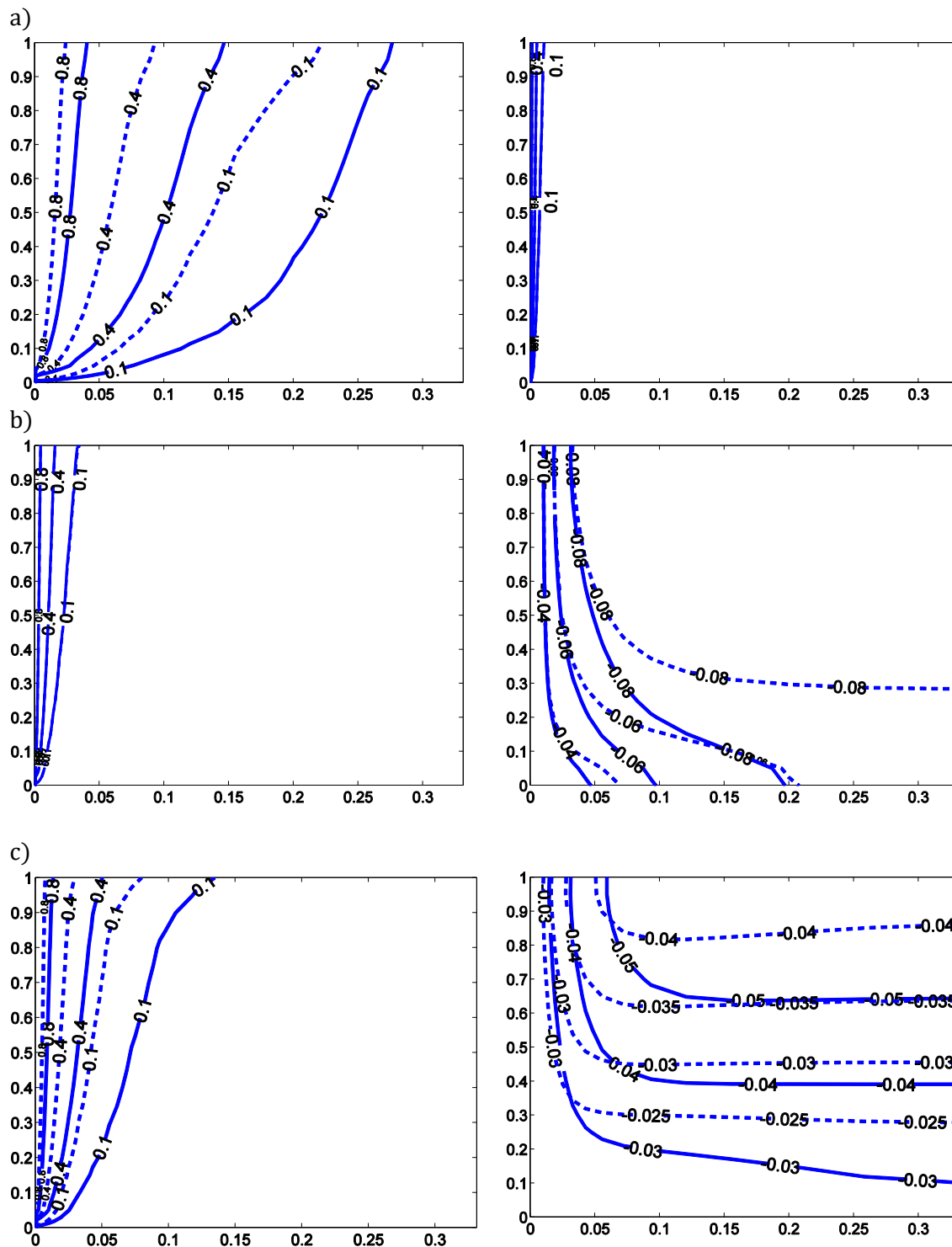


Figure 7: a) Isotherms, b) Isoconcentration and c) Streamlines

Left $Ra = 200$, Right $Ra = 2000$

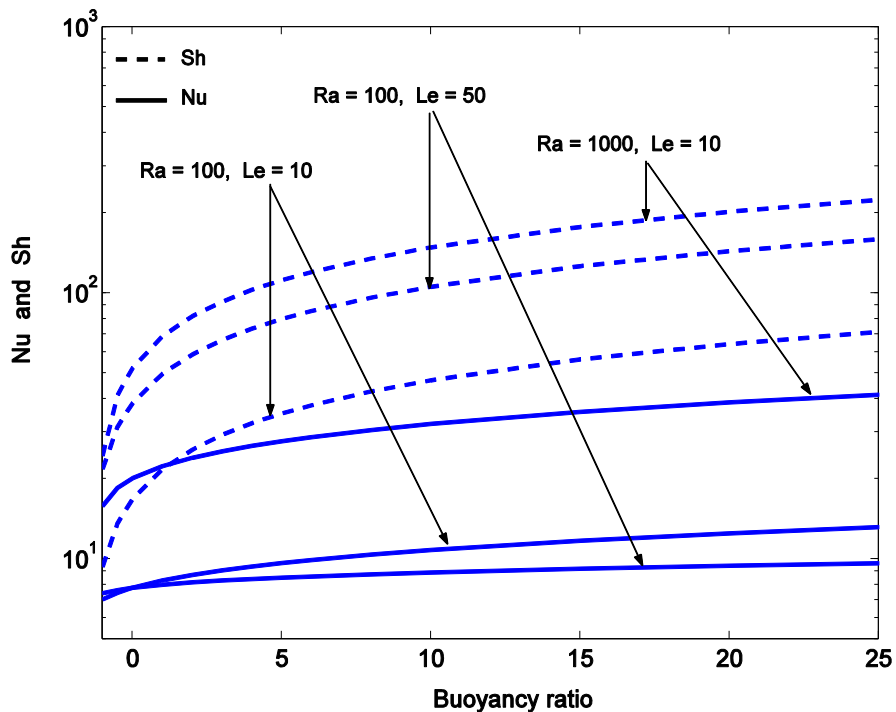


Figure 8: Nu and Sh for various values of N and Ra .

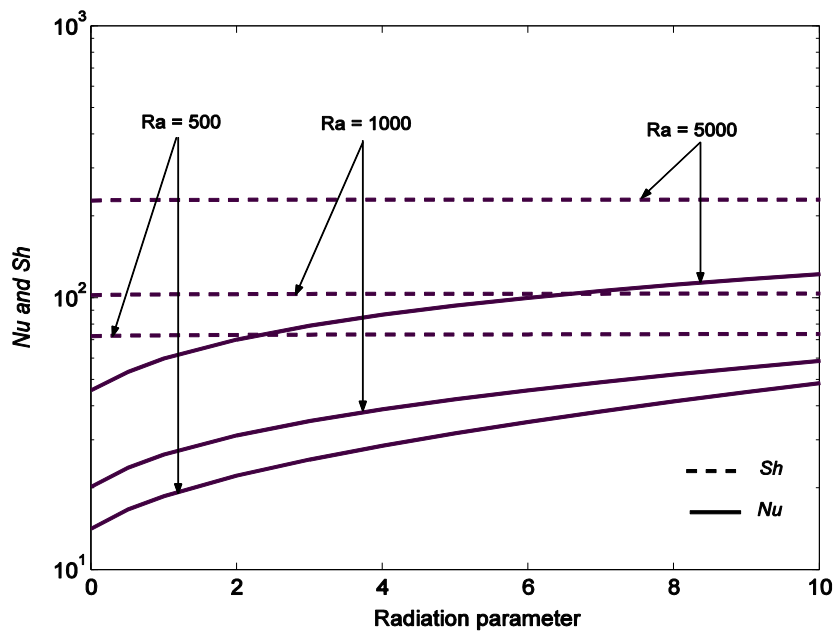


Figure 9: Nu and Sh for various values of Rd and Ra , at $Le = 10$, $N = 4$

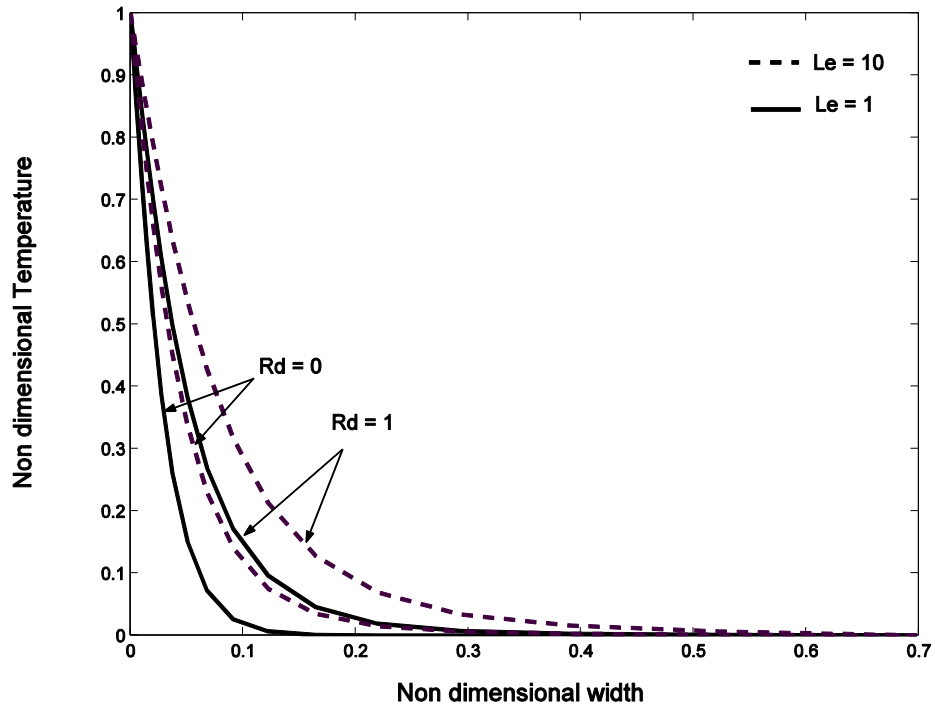


Figure 10: Temperature variation with Le at $Ra = 100$, $N = 4$

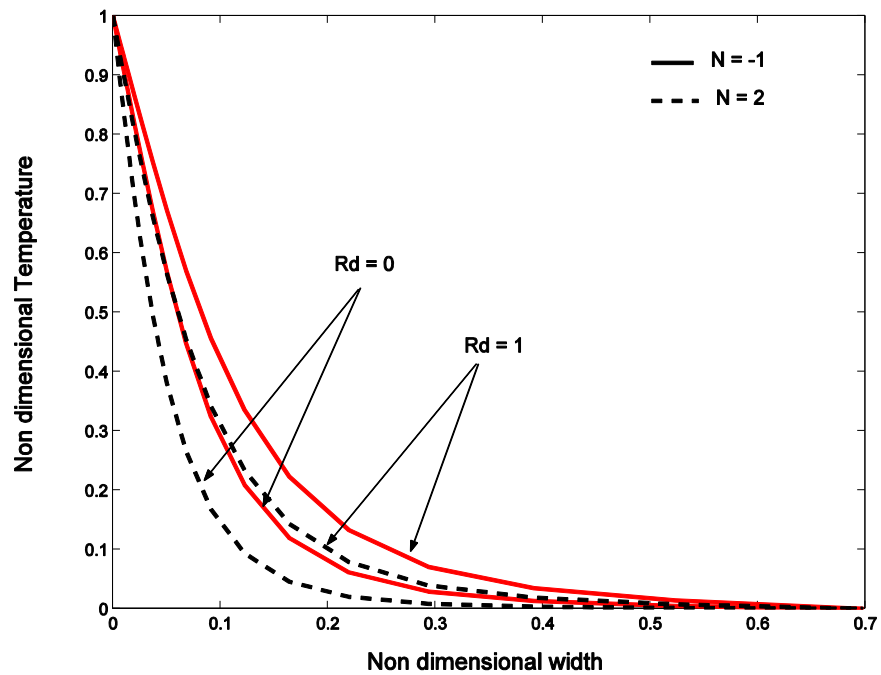


Figure 11: Temperature variation with N at $Ra = 100$, $Le = 10$

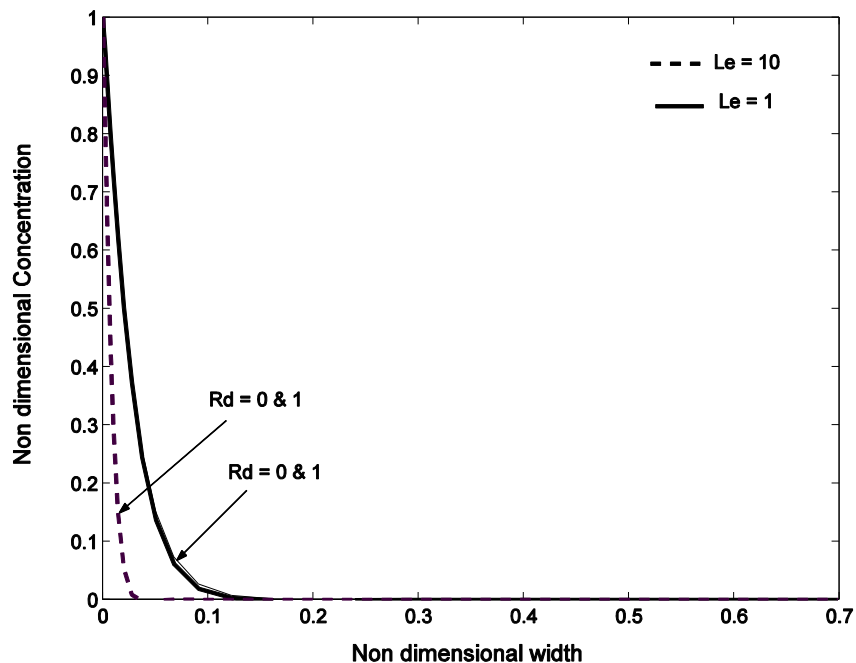


Figure 12 Concentration variation with Le at $Ra = 100$, $N = 4$

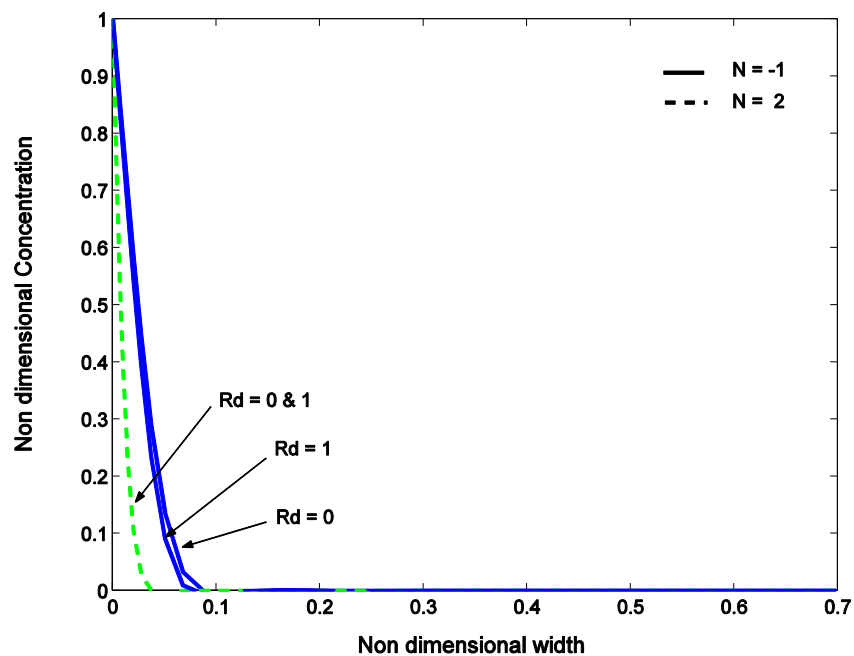


Figure 13 Concentration variation with N at $Ra = 100$, $Le = 10$

5. Conclusion

The heat and mass transfer in a saturated porous medium supported by vertical plate is analysed. FEM is used to solve the governing equations. Effect of various non-dimensional parameters on heat and fluid flow behavior is investigated. It is observed that the Nusselt number decreases and Sherwood number

increases due to increase in Lewis number. The impact of Lewis number is more on Sherwood number than on Nusselt number. The increased buoyancy ratio leads to increase in Nusselt and Sherwood numbers. Increase in Rayleigh number increases Nu and Sh . Radiation parameter affects Nu to greater extent than Sh . In general Nu and Sh increases with increased radiation parameter. The effect of Lewis number, Buoyancy ratio and Rayleigh number vanishes at higher radiation parameter. It is also observed that, at low radiation parameter convection dominate and at high radiation parameter the conduction is prominent. Non dimensional temperature gradient decreases at higher radiation parameter. Non-dimensional concentration gradient increases slightly with increase in radiation parameter.

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