

International Journal of Advanced Thermofluid Research

ISSN 2455-1368 www.ijatr.org



Geophysical Flow Simulation by using Riemann Solvers

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Keywords F-wave • HLLE • Roe • SWE • Well-balance.	Abstract Geophysical flow simulation for the prediction of emergency situations such as inundation, dam break, oil spills and tsunami is an active field of research. Depth-averaged form of conservation equations, known as shallow water equations (SWE), are the main inputs for such simulations. Recent research has proved that SWEs are powerful enough to capture up the most crucial parameters that define those emergency situations. In this research the non-linear system of SWEs are solved by using different approximate Riemann solvers and listed out their weaknesses. An advanced approach of treating source terms in a form of wave which satisfies the well-balancing condition is discussed and all the Riemann solvers used are compared with the benchmark test cases proposed by National Oceanic and Atmospheric Administration (NOAA) to prove their workability.		
Received Sep 26, 2016	Revised Nov 12, 2016	Accepted Nov 16, 2016	Published Dec 01, 2016
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DOI: https://doi.org/10.51141/IJATR.2016.2.1.6		© 2015 IREEE Press. All rights reserved.	

1. Introduction

Riemann solvers are well applied to hyperbolic conservation equations. The computational domain is split up into discrete volumes and each face is considered to be a discontinuity to solve a Riemann problem. Roe (1981)'s approach is a simple linearized Riemann solver, which is well applied to the field of gas dynamics; although it violates entropy at certain conditions, it can be fixed with various entropy fixes (Hudson, 1999). In case of shallow water equations (SWE), a new set of problem on well-balancing arises during steady state computation where the source terms are equally higher as the convective terms.

In this research three various Riemann solvers are being discussed and validated against benchmark test cases of analytical or experimental to find out the best possible solver at the current

stage. At first the standard Roe's Flux Difference Splitting (FDS) (Roe, 1981), which has its application on almost every hyperbolic system of equations, is discussed. Subsequently, LeVeque's f-wave approach (LeVeque, 2004) and an advanced f-wave type Augmented 4 Wave Scheme (A4WS) (George and LeVeque, 2006) are discussed.

2. Shallow Water Equations

SWEs are depth-averaged form of Navier-Strokes equation, which govern the flow phenomenon of long gravity waves; they have good application in geophysical flows such as tsunami waves, dam break and inundation of river or sea water. The main assumptions made while deriving the equations are: the fluid is considered to be inviscid and the vertical velocity is very less than the horizontal velocity (for further details about the derivation refer Dawson and Mirabito (2008)). The parameters governing the flow are described in the Fig. 1, where the height of the water column is h(x), the bathymetry measured from absolute zero is B(x) and the horizontal velocity is u(x).



Fig. 1. Variables governing the flow.

$$\frac{\partial q}{\partial t} + \frac{\partial F(q)}{\partial x} = R(x) \tag{1}$$

The vector form of conservative one-dimensional (1D) SWE is given in Eqn. (1), where the conservative variables are grouped in a vector q(x, t), the flux variables in a vector F(q) and the source terms which give raise to additional momentum are grouped in a vector R(x, q), as given by:

$$q(x,t) = \begin{bmatrix} h\\ uh \end{bmatrix}, F(q) = \begin{bmatrix} uh\\ hu^2 + \frac{1}{2}gh^2 \end{bmatrix}, \quad R(x,q) = \begin{bmatrix} 0\\ -ghB'(x) - \tau_x \end{bmatrix}$$
(2)

As we have derived inviscid SWE, the bottom friction term τ_x has to be added with the momentum equation through some empirical relations which will be discussed later. SWE can be written in 2D as:

$$\frac{\partial q}{\partial t} + \frac{\partial (F(q))}{\partial x} + \frac{\partial (G(q))}{\partial y} = R(x, y)$$
(3)

where
$$q(x,t) = \begin{bmatrix} h\\ hu\\ hv \end{bmatrix}$$
, $F(q) = \begin{bmatrix} hu\\ hu^2 + \frac{1}{2}gh^2\\ huv \end{bmatrix}$, $G(q) = \begin{bmatrix} hv\\ huv\\ hv^2 + \frac{1}{2}gh^2 \end{bmatrix}$, $R(x,q) = \begin{bmatrix} 0\\ -ghB'(x) - \tau_x\\ -ghB'(y) - \tau_y \end{bmatrix}$

3. Riemann Solvers

A. Roe FDS

Roe (1981) derived an approach which approximates systems of conservation laws by using a piecewise constant approximation.

I. Without Source Term

Considering the governing equation without the source component results in homogeneous system of equations, and discretising with FTCS explicit method gives:

$$\frac{(q^{n+1}-q^n)}{\Delta t} + \frac{\left(F_{i+\frac{1}{2}}^n - F_{i-\frac{1}{2}}^n\right)}{\Delta x} = 0$$
(4)

The fluxes at the faces are given as $F_{i+\frac{1}{2}}^{n}$ and $F_{i-\frac{1}{2}}^{n}$ which are determined as:

$$F_{i+\frac{1}{2}}^{n} = \frac{1}{2}(f_{i+1}^{n} + f_{i}^{n}) - \frac{1}{2}\left(\sum_{k=1}^{m} \tilde{\lambda}_{k} \tilde{e}_{k} \tilde{\alpha}_{k}\right)$$
(5)

For the stencil of grids with its index i+1 as right of the face and i as left of the face. The wave speed is taken as Roe speed $\tilde{\lambda}_k$ and the corresponding Eigen vector is \tilde{e}_k with its magnitude as $\tilde{\alpha}_k$

$$\widetilde{\lambda}_1 = \widetilde{u} + \widetilde{c}, \qquad \widetilde{\lambda}_2 = \widetilde{u} - \widetilde{c}$$

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Special Issue of Selected Papers from 2nd International Conference on Computational Methods in Engineering and Health Sciences (ICCMEH- 2015), 19-20 December 2015, Universiti Putra Malaysia, Selangor, Malaysia.

$$\widetilde{e}_{1} = \begin{bmatrix} 1\\ \widetilde{u} + \widetilde{c} \end{bmatrix}, \qquad \widetilde{e}_{2} = \begin{bmatrix} 1\\ \widetilde{u} - \widetilde{c} \end{bmatrix}$$
$$\widetilde{\alpha}_{1} = \frac{1}{2}\Delta h + \frac{1}{2\widetilde{c}}(\Delta(hu) - \widetilde{u}\Delta h), \qquad \widetilde{\alpha}_{2} = \frac{1}{2}\Delta h - \frac{1}{2\widetilde{c}}(\Delta(hu) - \widetilde{u}\Delta h)$$

where \tilde{u} and \tilde{c} are Roe averages given as:

$$\tilde{u} = \frac{\sqrt{h_r}u_r + \sqrt{h_l}u_l}{\sqrt{h_r} + \sqrt{h_l}} \text{ and } \tilde{c} = \sqrt{g\frac{h_r + h_l}{2}}$$

II. With Source Term

Source terms are added through a common operator type splitting scheme called as Strang Splitting (Strang, 1968). It involves computations with fractional time step ($\Delta t/2$) and complete time step (Δt) for source terms; the concept is to split the Eqn. (1) into a homogeneous PDE and an ODE.

$$\frac{\partial q}{\partial t} + \frac{\partial f(q)}{\partial x} = 0$$
$$\frac{\partial q}{\partial t} = R(x, q)$$

When the water depth (h) reduces than a certain level it exhibits transonic expansion, so simple entropy fix by Alcrudo et al. (1992) is followed. This allows us to work on nearly dry states of water depth as low as 1E-3.

B. F-wave Method

In earlier section of Roe's scheme approach the interface flux $(F_{i+\frac{1}{2}}^n)$ is determined to update the solution. Alternatively the structure of an approximate Riemann solution can be directly used to update the numerical solution. The effect of moving wave can be directly re-averaged into the computational grids. For instance at $x_{i-\frac{1}{2}}$ produces set of waves:

$$Q_i - Q_{i-1} = \sum_{p=1}^m W_{i-\frac{1}{2}}^p$$

Similarly the waves which carry the flux are known as flux waves or f-waves, which are given as the jump in flux between the two sides of the face as:

$$f(q_i) - f(q_{i-1}) = \sum_{1}^{p} Z_{i-\frac{1}{2}}^{p} = \sum_{1}^{p} \beta_{i-\frac{1}{2}}^{p} r_{i-\frac{1}{2}}^{p}$$
(6)

The vector $r_{i-\frac{1}{2}}^{p}$ and its corresponding wave-speeds are selected based on the structure of the PDE. From Eqn. (6), the difference in flux can be used to calculate $\beta_{i-\frac{1}{2}}^{p}$ by Camer's rule and then it is substituted back to estimate the f-wave $Z_{i-\frac{1}{2}}^{p}$. The discretised form involving the f-waves is given as:

$$q_{i,j}^{n+1} = q_{i,j}^{n} + \frac{\Delta t}{\Delta x} \left[\sum Z_{i-\frac{1}{2}}^{+} + \sum Z_{i+\frac{1}{2}}^{-} \right] + \frac{\Delta t}{\Delta y} \left[\sum Z_{j-\frac{1}{2}}^{+} + \sum Z_{j+\frac{1}{2}}^{-} \right]$$
(7)

 $Z_{i-\frac{1}{2}}^+$ are the waves which move in positive x-direction from left face $(x_{i-\frac{1}{2}})$ and enter the cell. similarly $Z_{i-\frac{1}{2}}^-$ are the waves which move in negative x-direction from right face $(x_{i+\frac{1}{2}})$ and enter the cell. $\sum Z_{i-\frac{1}{2}}^+$ and $\sum Z_{i-\frac{1}{2}}^-$ are called as updates as they represent the sum of waves that update the cell for the next time step. The wave speeds can be taken as the same as Roe speed used in previous approach, but it doesn't ensure depth positivity at certain places. So the modified speed suggested by Einfeldt (1988) for use with the HLLE solver is being used here.

$$\tilde{s}_{i-\frac{1}{2}}^{-} = \min(\lambda^{-}(q_{i-1}^{n}), \tilde{\lambda}_{i-\frac{1}{2}}^{-})$$
$$\tilde{s}_{i-\frac{1}{2}}^{+} = \max(\lambda^{+}(q_{i-1}^{n}), \tilde{\lambda}_{i-\frac{1}{2}}^{+})$$

 λ^- and λ^+ are the eigen values of the flux Jacobian, $\tilde{\lambda}_{i-\frac{1}{2}}^+$ and $\tilde{\lambda}_{i-\frac{1}{2}}^-$ are the Roe speeds discussed in the previous section.

$$\begin{split} s^{1}_{i-\frac{1}{2}}(h^{*}_{i-\frac{1}{2}}) &= u_{i-\frac{1}{2}} + 2\sqrt{gh_{i-\frac{1}{2}}} - 3\sqrt{gh^{*}}\\ s^{2}_{i-\frac{1}{2}}(h^{*}_{i-\frac{1}{2}}) &= u_{i+\frac{1}{2}} - 2\sqrt{gh_{i+\frac{1}{2}}} + 3\sqrt{gh^{*}} \end{split}$$

C. A4SW

Originally developed by George and LeVeque (2006), it make uses of the same f-wave propagation algorithm discussed earlier expect that an additional wave is added by augmentation. f-wave approach can simulate the source without balancing issues but it exhibits entropy violations which will be discussed later. For a system of n equations there are n characteristic waves; in certain cases such as a sonic point which leads to entropy violation, this method uses an additional wave called entropy wave, which fixes this violation. However, to do so, the system should have an additional equation; hence the momentum flux is augmented to the original system Eqn. (2), which allows the use of an additional wave namely entropy correction wave. The augmented system with momentum flux and bathymetry can be written as:

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Special Issue of Selected Papers from 2nd International Conference on Computational Methods in Engineering and Health Sciences (ICCMEH- 2015), 19-20 December 2015, Universiti Putra Malaysia, Selangor, Malaysia.

$$\tilde{q} = \begin{bmatrix} h \\ uh \\ hu^2 + \frac{1}{2}gh^2 \\ b \end{bmatrix}$$

I. Choosing wave speeds $(s_{i-\frac{1}{2}}^{p})$ and corresponding Vectors $(r_{i-\frac{1}{2}}^{p})$

For ID set of equations, we get two speeds from the original set of equations, and the first and third pair are related to them. We can name them as p = 1 and p = 3, from the Jacobian of SWE:

$$\{w^{\pm}q,\lambda^{\pm}(q)\} = \{(1,u \pm \sqrt{gh})^T, u \pm \sqrt{gh}\}$$

we choose:

$$\begin{cases} r_{i-\frac{1}{2}}^{1}, s_{i-\frac{1}{2}}^{1} \\ \end{cases} = \begin{cases} (1, \tilde{s_{i-\frac{1}{2}}}^{-}, (\tilde{s_{i-\frac{1}{2}}}^{-})^{2})^{T}, \tilde{s_{i-\frac{1}{2}}}^{-} \\ \end{cases} \\ \begin{cases} r_{i-\frac{1}{2}}^{3}, s_{i-\frac{1}{2}}^{3} \\ \end{cases} = \begin{cases} (1, \tilde{s_{i-\frac{1}{2}}}^{+}, (\tilde{s_{i-\frac{1}{2}}}^{+})^{2})^{T}, \tilde{s_{i-\frac{1}{2}}}^{+} \\ \end{cases} \end{cases}$$

 $s_{i-\frac{1}{2}}^{-}$ and $s_{i-\frac{1}{2}}^{+}$ are given as:

$$\tilde{s}_{i-\frac{1}{2}}^{-} = \min(\lambda^{-}(q_{i-1}^{n}), \tilde{\lambda}_{i-\frac{1}{2}}^{-})$$
$$\tilde{s}_{i-\frac{1}{2}}^{+} = \max(\lambda^{+}(q_{i}^{n}), \tilde{\lambda}_{i-\frac{1}{2}}^{+})$$

where λ , is the eigen value for Roe averaged Jacobian, and the speed *s* is referred to Einfeldt speeds.

II. Entropy Correction Wave

In the event of strong rarefaction in the first family of waves:

$$\left\{r_{i-\frac{1}{2}}^{2}, s_{i-\frac{1}{2}}^{2}\right\} = \left\{(1, s_{i-\frac{1}{2}}^{1}(h_{i-\frac{1}{2}}^{*}), (s_{i-\frac{1}{2}}^{1}(h_{i-\frac{1}{2}}^{*}))^{2}, 0)^{T}, s_{i-\frac{1}{2}}^{1}(h_{i-\frac{1}{2}}^{*})\right\}$$

In the event of strong rarefaction in the second family of waves:

$$\left\{r_{i-\frac{1}{2}}^{2}, s_{i-\frac{1}{2}}^{2}\right\} = \left\{(1, s_{i-\frac{1}{2}}^{2}(h_{i-\frac{1}{2}}^{*}), (s_{i-\frac{1}{2}}^{2}(h_{i-\frac{1}{2}}^{*}))^{2}, 0)^{T}, s_{i-\frac{1}{2}}^{2}(h_{i-\frac{1}{2}}^{*})\right\}$$

where,

$$\begin{split} s^{1}_{i-\frac{1}{2}}(h^{*}_{i-\frac{1}{2}}) &= u_{i-\frac{1}{2}} + 2\sqrt{gh_{i-\frac{1}{2}}} - 3\sqrt{gh^{*}}, \\ s^{2}_{i-\frac{1}{2}}(h^{*}_{i-\frac{1}{2}}) &= u_{i+\frac{1}{2}} - 2\sqrt{gh_{i+\frac{1}{2}}} + 3\sqrt{gh^{*}}, \end{split}$$

and h^* is the middle state depth given by HLLE middle state.

If any strong rarefaction is not present, it is enough to take as:

$$\left\{r_{i-\frac{1}{2}}^{2}, s_{i-\frac{1}{2}}^{2}\right\} = \left\{(0,0,1)^{T}, \frac{1}{2}(s_{i-\frac{1}{2}}^{-} + s_{i-\frac{1}{2}}^{+})\right\}$$

III. Including Source Term

The standard approach of fractional stepping to include the source term fails at preserving the required balance as stated earlier. Here the effect of the source term is included by introducing a fourth wave to the solver. Now the decomposition would look like:

$$\begin{bmatrix} h_i - h_{i-1} \\ (hu)_i - (hu)_{i-1} \\ \phi(q_i) - \phi(q_{i-1}) \\ B_i - B_{i-1} \end{bmatrix} = \sum_{p=1}^3 \beta_{i-\frac{1}{2}}^p w_{i-\frac{1}{2}}^p + \beta_{i-\frac{1}{2}}^0 w_{i-\frac{1}{2}}^0$$

where $\beta_{i-\frac{1}{2}}^{0} w_{i-\frac{1}{2}}^{0}$ is the steady state f-wave due to bathymetry addition. It can be written as $(B_i - B_{i-1})w_{i-\frac{1}{2}}^{0}$ if a smooth solution exists between two points in Shallow water equation. Now the equation can be rewritten with flux difference decomposition of characteristic waves only in right side and by moving the steady state wave to the left as:

$$\begin{bmatrix} h_i - h_{i-1} \\ (hu)_i - (hu)_{i-1} \\ \phi(q_i) - \phi(q_{i-1}) \\ B_i - B_{i-1} \end{bmatrix} - (B_i - B_{i-1}) w_{i-\frac{1}{2}}^0 = \sum_{p=1}^3 \beta_{i-\frac{1}{2}}^p w_{i-\frac{1}{2}}^p$$

More details about estimation of steady state wave $w_{i-\frac{1}{2}}^{0}$ can be found in the work of George and LeVeque (2006); β can be calculated by camer's rule similar to the calculation without source addition. With this calculated β the f-waves are estimated from Eqn. (6).

IV. Addition of Bottom Friction

The SWEs are derived on the assumption that the fluid is inviscid, which makes the velocity profile to be constant in vertical direction. But its validity decreases as the depth of the fluid decreases and velocity increases; these characters are mostly exhibited in inundation regimes. This motivates us to use an empirically derived friction term. In this research, the friction term is based on an

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empirically determined constant namely Manning coefficient (n), which ranges from n = 0.013 to n = 0.025 depending upon the bottom surface.

$$\begin{aligned} \tau_x &= \frac{gn^2}{h^{7/3}} hu \sqrt{(hu)^2 + (hv)^2} \\ \tau_y &= \frac{gn^2}{h^{7/3}} hv \sqrt{(hu)^2 + (hv)^2} \end{aligned}$$

These friction terms are explicitly added to Eqn. (7) through forward Euler as:

$$q_{i,j}^* = q_{i,j}^n + \frac{\Delta t}{\Delta x} \left[\sum_{i=\frac{1}{2}}^{+} + \sum_{i=\frac{1}{2}}^{-} \right] + \frac{\Delta t}{\Delta y} \left[\sum_{j=\frac{1}{2}}^{+} + \sum_{j=\frac{1}{2}}^{-} \right]$$
$$q_{i,j}^{n+1} = q_{i,j}^* - \Delta t \tau(q^*)$$

where $\tau(q^*)$ is given as:

$$\tau(q^*) = \begin{bmatrix} 0 \\ \tau_x(q^*) \\ \tau_y(q^*) \end{bmatrix}$$

4. Benchmark Test Problems

A. 1D Dam Break

The setup for computation is simple; the water height in half of the length is higher than the other, which creates a discontinuity at the middle. When the time step advances, there forms a shock and the expansion waves propagate in opposite directions.

B. 1D Solitary wave over a simple beach

This problem is an excellent test case for inclusion of source bathymetry, and drying and wetting shoreline propagation. Initially a solitary wave propagates over a constant depth and then over a sloping beach to reach the shoreline. This is just like a tidal wave hitting the beach except that the domain is over simplified. For more detailed description refer Synolakis et al. (2007).

C. Solitary wave over a composite beach

The wave travels on different piecewise linear bathymetry and hits a wall to get reflected back. There are various gauges placed at middle of the sloped bathymetry which measures the water level raise at certain time intervals.

D. 2D Radial Dam break

Water level is maintained at higher level at a circular region at the centre of the domain (depth = 2.5 m) and then the discontinuity propagates radially (Fig. 4).



Fig. 2 Schematic of the wave over a sloped beach.



Fig. 3 Schematic of the Composite beach.

E. Wave hits a 3D complex beach

The domain is taken as illustrated in Fig. 5. The time-dependent wave that enters the domain follows the profile as shown in Fig. 6.







Fig. 5 Bathymetry data of the 3D Beach.



Fig. 6 The time dependent wave that enters the domain.

5. Results and Discussion

Computations for the benchmark problems are performed with the solvers discussed in sections <u>3.A</u>, <u>3.B</u> and <u>3.C</u>. The benchmark problems that are discussed in <u>4.A</u> and <u>4.D</u> are compared with analytical solutions as they are simple and exact solutions are readily available. The problems discussed in <u>4.B</u>, <u>4.C</u> and <u>4.E</u> are compared with the experimental results obtained by gauges placed at various location inside the domain.

A. 1D Dam Break

Computations are performed till 0.1 sec with time step of 0.001 sec and the depth at t=0.1sec is plotted with the exact solution (Synolakis et al., 2007).The solutions are compared with fully wet condition (Fig. 7) and with dry state (Fig. 8). The solutions for fully wet condition by all the tree schemes agree well with the exact solution. In dry dam break case the f-wave scheme diverges (Fig. 9) as it doesn't satisfy the entropy condition. This is where A4WS gains advantage over f-wave by making use of the additional entropy wave.

B. 1D Solitary wave over a simple beach

The computations are compared with the experimental results (Synolakis et al., 2007) and found that A4WS exhibits better wetting and drying with less computational time (Fig. 10).

C. Solitary wave over a composite beach

In this case f-wave and A4WS share similar results; again the computational results are compared with the experimental results (Synolakis et al., 2007) at various gauge locations (refer Synolakis et al. (2007) for gauge locations) and found in good agreement.



Fig. 7 Water level at t = 0.1 sec. (fully wet)



Fig. 8 Water level at t = 0.1 sec. (dry state at the right)



Fig. 9 f-wave solution at Entropy Violation.



Fig. 10. Wave propagation at various non-dimensional times.



Fig. 11. Water level raise at various gauge locations.

D. 2D Radial Dam break

Both wet and dry case scenarios are computed and compared with a high-resolution scheme (Liang et al., 2004). Fully dry case (Fig. 13) experiences only one characteristic family of expansion fan towards the higher depth region.

A. Wave hits a 3D complex beach

From the input wave (Fig. 6) it can be seen that for a time period of first 3 to 4 sec the domain has to be simulated in a rest state; this rest sate causes balancing issues as the momentum due to bathymetry is considerably larger than the convection.

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Fig. 12. Wave propagation of Wet case at various times.



Fig. 13. Wave propagation of Dry case at various times.

A4WS perfectly balances it as the momentum-raise due to source term is treated in terms of wave in a stationary form whereas the Roe (FDS) fails to simulate such a problem. The computational results are compared with the experimental results (Synolakis et al., 2007) at various gauge locations and found in good match.



Fig. 14. Water level raise at various gauge locations.

6. Conclusion

The advanced A4WS solver and its capabilities have been explored. Roe (FDS) suffers from wellbalancing and the splitting scheme reduces its CFL which in turn increases the computation time. Fwave suffers from entropy violation and prevents it to work on dry states. Further, A4WS can be extended to unstructured and dynamically varying grids for large scale Tsunami problems.

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